Propagating Soft Table Constraints

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with

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CP'2012 - October 12, 2012

Weighted Constraint Satisfaction Problem

- Soft constraints and forbidden cost k

$c_\emptyset$
0

c_{xz}		
x	z	$cost$
a	a	1
d	a	2
c	b	0
b	d	k

→ forbidden tuple

c_x	
x	$cost$
c	0
d	1

- Solve WCSP: find a complete instantiation with minimal cost

Weighted Constraint Satisfaction Problem

Soft table constraint

	w	x	y	z	costs
τ_0	a	c	a	c	2
τ_1	a	c	b	a	0
τ_2	b	b	a	b	3
τ_3	b	c	b	a	0
τ_4	c	a	a	b	1
τ_5	c	b	c	c	1

table — *costs*

k — *defaultCost*

explicit tuples (table, costs) — implicit tuples (defaultCost)

Consistencies for WCSP

- Soft arc consistencies inherited from CSP: NC*, AC*, Generalized Arc Consistency (GAC),...
- Sophisticated soft arc consistencies have been proposed too: FDAC, EDAC, VAC, OSAC,...
- Cost transfer operations (project, unary project,...) performed to enforce consistencies

Motivations

Drawback

Cost transfer algorithms particularly efficient to solve real-world binary or ternary problem instances ...

... but not efficient/adapted for large arity problem instances !

Proposition

Algorithm to enforce a weak version of GAC by combining:

- Simple Tabular Reduction
- cost transfer

Target:

- problem instances containing soft table constraints of large arity with default cost $= 0$ or k

GAC^w

Definition

The *extended* cost of a tuple τ on a soft constraint c_S, called $ecost(c_S, \tau)$, includes:

- cost of τ on c_S
- unary costs for τ of the variables in scope of c_S
- nullary cost $c_\emptyset$

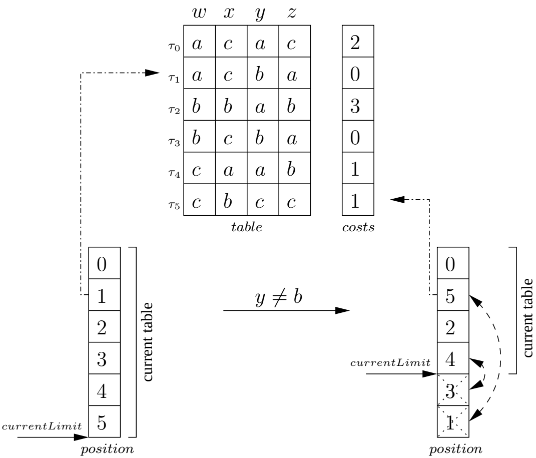
Definition

A value (x, a) is GAC^w-consistent on a soft constraint c_S iff there exists a tuple τ in c_S containing (x, a) with cost 0 and $ecost(c_S, \tau) < k$

A soft constraint c_S is GAC^w-consistent iff every value of c_S is GAC^w-consistent.

Weighted Simple Tabular Reduction (WSTR)

Maintains dynamically the list of allowed tuples in constraint tables



Algorithm data structures

- **position**, **currentLimit**: STR structures
- $\mathbf{c_1[x][a]}$: cost for (x, a) on unary constraint of x
- $\mathbf{minCosts[c_S][x][a]}$: minimal cost on c_S of tuples containing (x, a)
- $\mathbf{S^{sup}}$: uninstancied variables whose domain contains each a least one value for wich a support has not yet been found

Algorithm principle

- Step 1: find minimal costs for values
- Step 2: remove GAC^w-inconsistent values
- Step 3: find supports for values

Step 1: find minimal costs for values

$c_\emptyset$ = 0 k = 5

event $x \neq a$

c_1

	a	b	c
w	0	1	0
x	0	0	0
y	0	0	0
z	0	1	0

c_{wxyz}

position		w	x	y	z	costs
1	τ_0	a	c	a	c	2
4	τ_1	a	c	b	a	0
5	τ_2	b	b	a	b	3
2	τ_3	b	c	b	a	0
0	τ_4	c	a	a	b	1
3	τ_5	c	b	c	c	1
→ 6	τ_6	c	c	b	c	0

minCosts

	a	b	c
w	k	k	k
x	☒	k	k
y	k	k	k
z	k	k	k

$S^{sup} = \{w, x, y, z\}$

Step 1: find minimal costs for values

$c_\emptyset$ = 0 k = 5

event $x \neq a$

	a	b	c
w	0	1	0
x	0	0	0
y	0	0	0
z	0	1	0

c_1

position		w	x	y	z	*costs*
1	τ_0	a	c	a	c	2
4	τ_1	a	c	b	a	0
5	τ_2	b	b	a	b	3
2	τ_3	b	c	b	a	0
0	τ_4	c	a	a	b	1
3	τ_5	c	b	c	c	1
→ 6	τ_6	c	c	b	c	0

table

c_{wxyz}

	a	b	c
w	k	k	k
x	☒	k	k
y	k	k	k
z	k	k	k

minCosts

$S^{sup} = \{w, x, y, z\}$

Step 1: find minimal costs for values

$c_\emptyset$ 0 k 5

event $x \neq a$

	a	b	c
w	0	1	0
x	0	0	0
y	0	0	0
z	0	1	0

c_1

position		w	x	y	z	*costs*
1	τ_0	a	c	a	c	2
4	τ_1	a	c	b	a	0
5	τ_2	b	b	a	b	3
2	τ_3	b	c	b	a	0
0	τ_4	c	a	a	b	1
3	τ_5	c	b	c	c	1
→ 6	τ_6	c	c	b	c	0

position *table* *costs*

c_{wxyz}

	a	b	c
w	k	k	k
x		k	k
y	k	k	k
z	k	k	k

minCosts

$S^{sup} = \{w, x, y, z\}$

$\xrightarrow{\tau_1}$

τ_1 is VALID:

$\forall$ variable $x \in S$, $\tau_1[x] \in dom(x)$

τ_1 is ALLOWED (GAC^w-consistency):

$c_\emptyset \bigoplus_{x \in S} c_1(\tau_1[x]) + costs[\tau_1] \leq k \ (0 \leq 5)$

Step 1: find minimal costs for values

$c_\emptyset$ 0 k 5

event $x \neq a$

	a	b	c
w	0	1	0
x	0	0	0
y	0	0	0
z	0	1	0

c_1

position		w	x	y	z	*costs*
1	τ_0	a	c	a	c	2
4	τ_1	a	c	b	a	0
5	τ_2	b	b	a	b	3
2	τ_3	b	c	b	a	0
0	τ_4	c	a	a	b	1
3	τ_5	c	b	c	c	1
→ 6	τ_6	c	c	b	c	0

c_{wxyz}

minCosts (before):

	a	b	c
w	k	k	k
x	×	k	k
y	k	k	k
z	k	k	k

$\xrightarrow{\tau_1}$

minCosts (after):

	a	b	c
w	0	k	k
x	×	k	0
y	k	0	k
z	0	k	k

$S^{sup} = \{w, x, y, z\}$

Step 1: find minimal costs for values

$c_\emptyset$ = 0 k = 5

event $x \neq a$

	a	b	c
w	0	1	0
x	0	0	0
y	0	0	0
z	0	1	0

c_1

position		w	x	y	z	*costs*
1	τ_0	a	c	a	c	2
4	τ_1	a	c	b	a	0
5	τ_2	b	b	a	b	3
2	τ_3	b	c	b	a	0
0	τ_4	c	a	a	b	1
3	τ_5	c	b	c	c	1
→ 6	τ_6	c	c	b	c	0

c_{wxyz}

minCosts:

	a	b	c
w	k	k	k
x	☒	k	k
y	k	k	k
z	k	k	k

$\xrightarrow{\tau_1}$ *minCosts*:

	a	b	c
w	0	k	k
x	☒	k	0
y	k	0	k
z	0	k	k

$\xrightarrow{\tau_4}$ τ_4 is not VALID: $a \notin \text{dom}(x)$

$S^{sup} = \{w, x, y, z\}$

Step 1: find minimal costs for values

$c_\emptyset$ = 0 k = 5

event $x \neq a$

	a	b	c
w	0	1	0
x	0	0	0
y	0	0	0
z	0	1	0

c_1

position		w	x	y	z	*costs*
1	τ_0	a	c	a	c	2
6	τ_1	a	c	b	a	0
5	τ_2	b	b	a	b	3
2	τ_3	b	c	b	a	0
0	τ_4	c	a	a	b	1
3	τ_5	c	b	c	c	1
4	τ_6	c	c	b	c	0

position, *table*, *costs*

c_{wxyz}

	a	b	c
w	k	k	k
x	☒	k	k
y	k	k	k
z	k	k	k

minCosts

$\xrightarrow{\tau_1}$

	a	b	c
w	0	k	k
x	☒	k	0
y	k	0	k
z	0	k	k

minCosts

$\xrightarrow{\tau_4}$ τ_4 is not VALID: $a \notin \text{dom}(x)$

$S^{sup} = \{w, x, y, z\}$

Step 1: find minimal costs for values

$c_\emptyset$ = 0 k = 5

event $x \neq a$

	a	b	c
w	0	1	0
x	0	0	0
y	0	0	0
z	0	1	0

c_1

position		w	x	y	z	*costs*
1	τ_0	a	c	a	c	2
6	τ_1	a	c	b	a	0
5	τ_2	b	b	a	b	3
2	τ_3	b	c	b	a	0
0	τ_4	c	a	a	b	1
→ 3	τ_5	c	b	c	c	1
4	τ_6	c	c	b	c	0

position, *table*, *costs*

c_{wxyz}

minCosts

	a	b	c
w	k	k	k
x	X	k	k
y	k	k	k
z	k	k	k

$\xrightarrow{\tau_1}$ *minCosts*

	a	b	c
w	0	k	k
x	X	k	0
y	k	0	k
z	0	k	k

$\xrightarrow{\tau_6}$ *minCosts*

	a	b	c
w	0	k	0
x	X	k	0
y	k	0	k
z	0	k	0

$\xrightarrow{\tau_5}$ *minCosts*

	a	b	c
w	0	k	0
x	X	1	0
y	k	0	1
z	0	k	0

$S^{sup} = \{w, x, y, z\}$

Step 1: find minimal costs for values

$c_\emptyset$ = 0 k = 5

event $x \neq a$

	a	b	c
w	0	1	0
x	0	0	0
y	0	0	0
z	0	1	0

c_1

position		w	x	y	z	costs
1	τ_0	a	c	a	c	2
6	τ_1	a	c	b	a	0
5	τ_2	b	b	a	b	3
2	τ_3	b	c	b	a	0
0	τ_4	c	a	a	b	1
→ 3	τ_5	c	b	c	c	1
4	τ_6	c	c	b	c	0

position, table, costs

c_{wxyz}

τ_2 is not ALLOWED: $5 \geq k$

	a	b	c
w	k	k	k
x	⊠	k	k
y	k	k	k
z	k	k	k

minCosts

$\xrightarrow{\tau_1}$

	a	b	c
w	0	k	k
x	⊠	k	0
y	k	0	k
z	0	k	k

minCosts

$\xrightarrow{\tau_6}$

	a	b	c
w	0	k	0
x	⊠	k	0
y	k	0	k
z	0	k	0

minCosts

$\xrightarrow{\tau_5}$

	a	b	c
w	0	k	0
x	⊠	1	0
y	k	0	1
z	0	k	0

minCosts

↑ τ_2

$S^{sup} = \{w, x, y, z\}$

Step 1: find minimal costs for values

$c_\emptyset$ = 0 $\quad k$ = 5

event $x \neq a$

c_1:

	a	b	c
w	0	1	0
x	0	0	0
y	0	0	0
z	0	1	0

c_{wxyz}:

position		w	x	y	z	costs
1	τ_0	a	c	a	c	2
6	τ_1	a	c	b	a	0
5	τ_2	b	b	a	b	3
3	τ_3	b	c	b	a	0
0	τ_4	c	a	a	b	1
2	τ_5	c	b	c	c	1
4	τ_6	c	c	b	c	0

minCosts (initial):

	a	b	c
w	k	k	k
x	╳	k	k
y	k	k	k
z	k	k	k

$\xrightarrow{\tau_1}$ *minCosts*:

	a	b	c
w	0	k	k
x	╳	k	0
y	k	0	k
z	0	k	k

$\xrightarrow{\tau_6}$ *minCosts*:

	a	b	c
w	0	k	0
x	╳	k	0
y	k	0	k
z	0	k	0

$\xrightarrow{\tau_5}$ *minCosts*:

	a	b	c
w	0	k	0
x	╳	1	0
y	k	0	1
z	0	k	0

$\uparrow \tau_2$: τ_2 is not ALLOWED: $5 \geq k$

$S^{sup} = \{w, x, y, z\}$

Step 1: find minimal costs for values

$c_\emptyset$ = 0 k = 5

event $x \neq a$

c_1:

	a	b	c
w	0	1	0
x	0	0	0
y	0	0	0
z	0	1	0

C_{wxyz}:

position		w	x	y	z	costs
1	τ_0	a	c	a	c	2
6	τ_1	a	c	b	a	0
5	τ_2	b	b	a	b	3
3	τ_3	b	c	b	a	0
→ 0	τ_4	c	a	a	b	1
2	τ_5	c	b	c	c	1
4	τ_6	c	c	b	c	0

minCosts (initial):

	a	b	c
w	k	k	k
x	×	k	k
y	k	k	k
z	k	k	k

$S^{sup} = \{w, x, y, z\}$

→ τ_1 → *minCosts*:

	a	b	c
w	0	k	k
x	×	k	0
y	k	0	k
z	0	k	k

→ τ_6 → *minCosts*:

	a	b	c
w	0	k	0
x	×	k	0
y	k	0	k
z	0	k	0

→ τ_5 → *minCosts*:

	a	b	c
w	0	k	0
x	×	1	0
y	k	0	1
z	0	k	0

→ τ_3 →

	a	b	c
w	0	0	0
x	×	1	0
y	k	0	1
z	0	k	0

→ τ_0 →

	a	b	c
w	0	0	0
x	×	1	0
y	2	0	1
z	0	k	0

Step 1: find minimal costs for values (default cost = 0)

$c_\emptyset$ = 0 k = 5

$defaultCost$ = 0

	a	b	c
w	0	1	0
x	0	0	0
y	0	0	0
z	0	1	0

c_1

	a	b	c
w	0	0	0
x	×	1	0
y	2	0	1
z	0	k	0

$minCosts$

$S^{sup} = \{w, x, y, z\}$

$position$		w	x	y	z	$costs$
1	τ_0	a	c	a	c	2
6	τ_1	a	c	b	a	0
5	τ_2	b	b	a	b	3
3	τ_3	b	c	b	a	0
→ 0	τ_4	c	a	a	b	1
2	τ_5	c	b	c	c	1
4	τ_6	c	c	b	c	0

$table$

c_{wxyz}

$dom(x) = \{b, c\}$

$dom(w) = dom(y) = dom(z) = \{a, b, c\}$

Step 1: find minimal costs for values (default cost = 0)

$c_\emptyset$ = 0 k = 5

$defaultCost$ = 0

	a	b	c
w	0	1	0
x	0	0	0
y	0	0	0
z	0	1	0

c_1

	a	b	c
w	0	0	0
x	X	1	0
y	2	0	1
z	0	k	0

$minCosts$

$S^{sup} = \{w, x, y, z\}$

position		w	x	y	z	costs
1	τ_0	a	c	a	c	2
6	τ_1	a	c	b	a	0
5	τ_2	b	b	a	b	3
3	τ_3	b	c	b	a	0
→ 0	τ_4	c	a	a	b	1
2	τ_5	c	b	c	c	1
4	τ_6	c	c	b	c	0

c_{wxyz} (position, table, costs)

$dom(x) = \{b, c\}$

$dom(w) = dom(y) = dom(z) = \{a, b, c\}$

number of valid explicit tuples : 1

Step 1: find minimal costs for values (default cost = 0)

$c_\emptyset$ 0 k 5

$defaultCost$ 0

	a	b	c
w	0	1	0
x	0	0	0
y	0	0	0
z	0	1	0

c_1

	a	b	c
w	0	0	0
x		1	0
y	2	0	1
z	0	k	0

$minCosts$

$S^{sup} = \{w, x, y, z\}$

position		w	x	y	z	costs
1	τ_0	a	c	a	c	2
6	τ_1	a	c	b	a	0
5	τ_2	b	b	a	b	3
3	τ_3	b	c	b	a	0
→ 0	τ_4	c	a	a	b	1
2	τ_5	c	b	c	c	1
4	τ_6	c	c	b	c	0

$position$ $table$ $costs$

c_{wxyz}

$dom(x) = \{b, c\}$

$dom(w) = dom(y) = dom(z) = \{a, b, c\}$

number of valid explicit tuples : 1

$|\Pi_{y \in S | y \neq x} dom(y)|$: 27

Step 1: find minimal costs for values (default cost = 0)

$c_\emptyset$ = 0 k = 5

$defaultCost$ = 0

	a	b	c
w	0	1	0
x	0	0	0
y	0	0	0
z	0	1	0

c_1

	a	b	c
w	0	0	0
x	☒	0	0
y	2	0	1
z	0	k	0

$minCosts$

$S^{sup} = \{w, x, y, z\}$

position		w	x	y	z	costs
1	τ_0	a	c	a	c	2
6	τ_1	a	c	b	a	0
5	τ_2	b	b	a	b	3
3	τ_3	b	c	b	a	0
→ 0	τ_4	c	a	a	b	1
2	τ_5	c	b	c	c	1
4	τ_6	c	c	b	c	0

$table$

c_{wxyz}

$dom(x) = \{b, c\}$

$dom(w) = dom(y) = dom(z) = \{a, b, c\}$

number of valid explicit tuples : 1

$|\Pi_{y \in S | y \neq x} dom(y)|$: 27

implicit tuples must be considered !

Step 2: remove GAC^w-inconsistent values

GAC^w-consistent ?

	a	b	c
w	0	0	0
x	☒	1	0
y	2	0	1
z	0	k	0

$minCosts$

$S^{sup} = \{w, x, y, z\}$

dom(w)= $\{a, b, c\}$
dom(x)= $\{b, c\}$
dom(y)= $\{a, b, c\}$
dom(z)= $\{a, b, c\}$

Step 2: remove GAC^w-inconsistent values

GAC^w-consistent ?

$ecost(c_S, \tau) < k$?

	a	b	c
w	0	0	0
x	X	1	0
y	2	0	1
z	0	k	0

$minCosts$

	a	b	c
w	0	0	0
x	X	1	0
y	2	0	1
z	0	X	0

$minCosts$

$S^{sup} = \{w, x, y, z\}$

$S^{sup} = \{w, x, y, z\}$

dom(w)= $\{a, b, c\}$
dom(x)= $\{b, c\}$
dom(y)= $\{a, b, c\}$
dom(z)= $\{a, b, c\}$

dom(w)= $\{a, b, c\}$
dom(x)= $\{b, c\}$
dom(y)= $\{a, b, c\}$
dom(z)= $\{a, b, c\}$

Step 2: remove GAC^w-inconsistent values

GAC^w-consistent ?

$ecost(c_S, \tau) < k$? → $c_S(\tau) = 0$?

	a	b	c
w	0	0	0
x	X	1	0
y	2	0	1
z	0	k	0

$minCosts$

$S^{sup} = \{w, x, y, z\}$

dom(w)= $\{a, b, c\}$
dom(x)= $\{b, c\}$
dom(y)= $\{a, b, c\}$
dom(z)= $\{a, b, c\}$

	a	b	c
w	0	0	0
x	X	1	0
y	2	0	1
z	0	X	0

$minCosts$

$S^{sup} = \{w, x, y, z\}$

dom(w)= $\{a, b, c\}$
dom(x)= $\{b, c\}$
dom(y)= $\{a, b, c\}$
dom(z)= $\{a, c\}$

	a	b	c
w	0	0	0
x	X	1	0
y	2	0	1
z	0	X	0

$minCosts$

$S^{sup} = \{w, x, y, z\}$

dom(w)= $\{a, b, c\}$
dom(x)= $\{b, c\}$
dom(y)= $\{a, b, c\}$
dom(z)= $\{a, c\}$

Step 3: find supports for values

	a	b	c
w	0	1	0
x	0	0	0
y	0	0	0
z	0	1	0

c_1

	a	b	c
w	0	0	0
x	×	1	0
y	2	0	1
z	0	×	0

$minCosts$

$$S^{sup} = \{x, y\}$$

Step 3: find supports for values

	a	b	c
w	0	1	0
x	0	1	0
y	0	0	0
z	0	1	0

c_1

project$(x, b, 1)$

	a	b	c
w	0	0	0
x	X	1	0
y	2	0	1
z	0	X	0

$minCosts$

$$S^{sup} = \{x, y\}$$

Step 3: find supports for values

	a	b	c
w	0	1	0
x	0	1	0
y	0	0	0
z	0	1	0

c_1

project$(x, b, 1)$

	a	b	c
w	0	0	0
x	×	1	0
y	2	0	1
z	0	×	0

$minCosts$

project$(y, a, 2)$

	a	b	c
w	0	1	0
x	0	1	0
y	2	0	0
z	0	1	0

c_1

$$S^{sup} = \{x, y\}$$

Step 3: find supports for values

	a	b	c
w	0	1	0
x	0	1	0
y	0	0	0
z	0	1	0

c_1

project$(y, a, 2)$

	a	b	c
w	0	1	0
x	0	1	0
y	2	0	0
z	0	1	0

c_1

	a	b	c
w	0	0	0
x	×	1	0
y	2	0	1
z	0	×	0

minCosts

project$(x, b, 1)$

	a	b	c
w	0	1	0
x	0	1	0
y	2	0	1
z	0	1	0

c_1

project$(y, c, 1)$

$$S^{sup} = \{x, y\}$$

Step 3: find supports for values

	a	b	c
w	0	1	0
x	0	1	0
y	0	0	0
z	0	1	0

c_1

project$(y, a, 2)$

	a	b	c
w	0	1	0
x	0	1	0
y	2	0	0
z	0	1	0

c_1

	a	b	c
w	0	0	0
x	×	1	0
y	2	0	1
z	0	×	0

minCosts

project$(x, b, 1)$

GAC^w-consistency established !

	a	b	c
w	0	1	0
x	0	1	0
y	2	0	1
z	0	1	0

c_1

project$(y, c, 1)$

	a	b	c
w	0	0	0
x	×	0	0
y	0	0	0
z	0	×	0

minCosts

dom(w)=dom(y)= $\{a, b, c\}$
dom(x)= $\{b, c\}$
dom(z)= $\{a, c\}$

$S^{sup} = \{x, y\}$

$S^{sup} = \{\}$

Number of solved instances per series

Series	#Inst	PFC-MRDAC- WSTR	PFC-MRDAC- GEN	Maintaining- GAC^w-WSTR	Maintaining- AC*	Maintaining- FDAC
crossoft-herald	50	33	10	**47**	11	11
crossoft-puzzle	22	**22**	9	**22**	18	18
crossoft-vg	64	**14**	6	**14**	7	7
rand-3	48	20	29	20	**32**	30
rand-10	20	**20**	0	**20**	0	0
ergo	19	13	10	15	15	**17**
linkage	30	0	0	0	1	**9**

Number of instances by series solved before time-out (1,200 seconds)

Conclusion and future work

- weak generalized arc consistency GAC^w on soft table constraints
- simple tabular reduction and cost transfer operations
- applied to any soft table constraint with a default cost of either 0 or k (large proportion of practical instances)
- efficient approach when soft table constraints have large arity

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Thank you for your attention !

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