

# **The Semigroups of Order 10**

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# The problem

enumerate the non-equivalent semigroups of order 10

- fill in a  $10 \times 10$  grid with the numbers 0 to 9
- must satisfy a binary associative relation
- search space  $10^{100}$

# The problem

$\odot$	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>	<b>5</b>	<b>6</b>	<b>7</b>	<b>8</b>	<b>9</b>
<b>0</b>	0	0	0	0	4	4	0	0	4	4
<b>1</b>	0	1	0	0	4	4	0	0	4	4
<b>2</b>	2	2	2	2	5	5	2	2	5	5
<b>3</b>	2	2	2	3	5	5	2	2	5	5
<b>4</b>	0	0	0	0	4	4	4	4	0	0
<b>5</b>	2	2	2	2	5	5	5	5	2	2
<b>6</b>	0	0	2	2	4	5	6	7	8	9
<b>7</b>	0	0	2	2	4	5	7	6	9	8
<b>8</b>	2	2	0	0	5	4	8	9	7	6
<b>9</b>	2	2	0	0	5	4	9	8	6	7

# The approach

- mathematical enumeration formulae
- constraint search

# The approach

- mathematical enumeration formulae  $\gg 99.9\%$
- constraint search  $\ll 0.1\%$

# Constraint search

- only interested in non-equivalent solutions → break  $2 \times 10!$  symmetries
- decomposition into several CSPs
- distributed search through model splitting

# Minion performance improvements

- remove entailed constraints
- improved lex propagation (QuickLex)
- watched literals

# Distributed search

- stop search and create new models with additional constraints that split the search space
- enables distribution of the models and fault-tolerance



# Computational effort

approx. 130 CPU years across 2 clouds and 2 clusters

**12,418,001,077,381,302,684**

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**Thank you!**