

Revisiting Neighborhood Inverse Consistency on Binary CSPs

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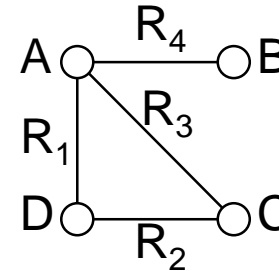
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Outline

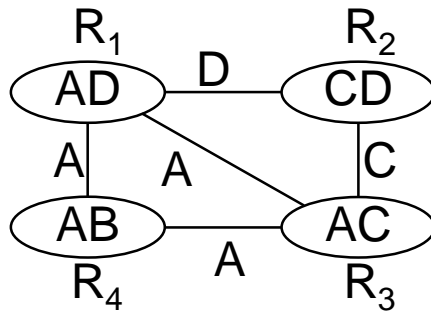
- Introduction: Relational NIC
- Structure of the dual graph of a binary CSP affects RNIC
- RNIC versus NIC, sCDC on binary CSPs
- Experimental results
- Conclusion

Constraint Satisfaction Problem

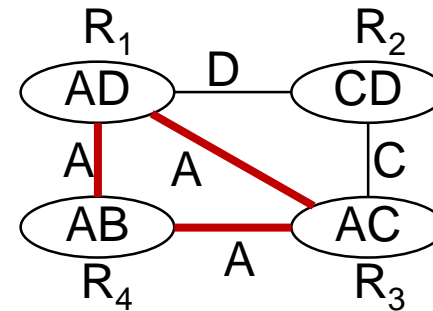
- Graphical Representation
 - Constraint graph
 - Dual graph
 - Minimal dual graph



Constraint graph



Dual graph

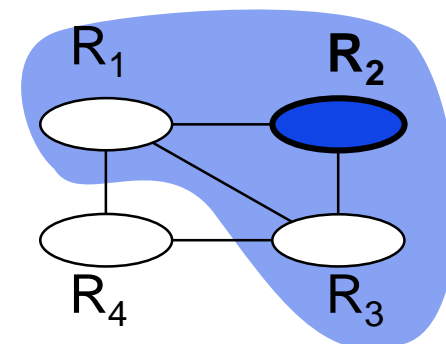
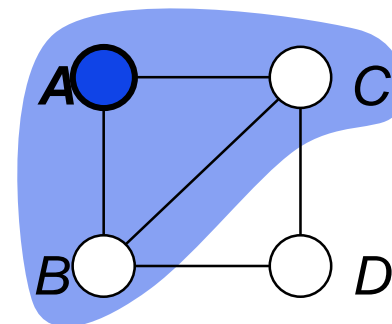


Minimal dual graph

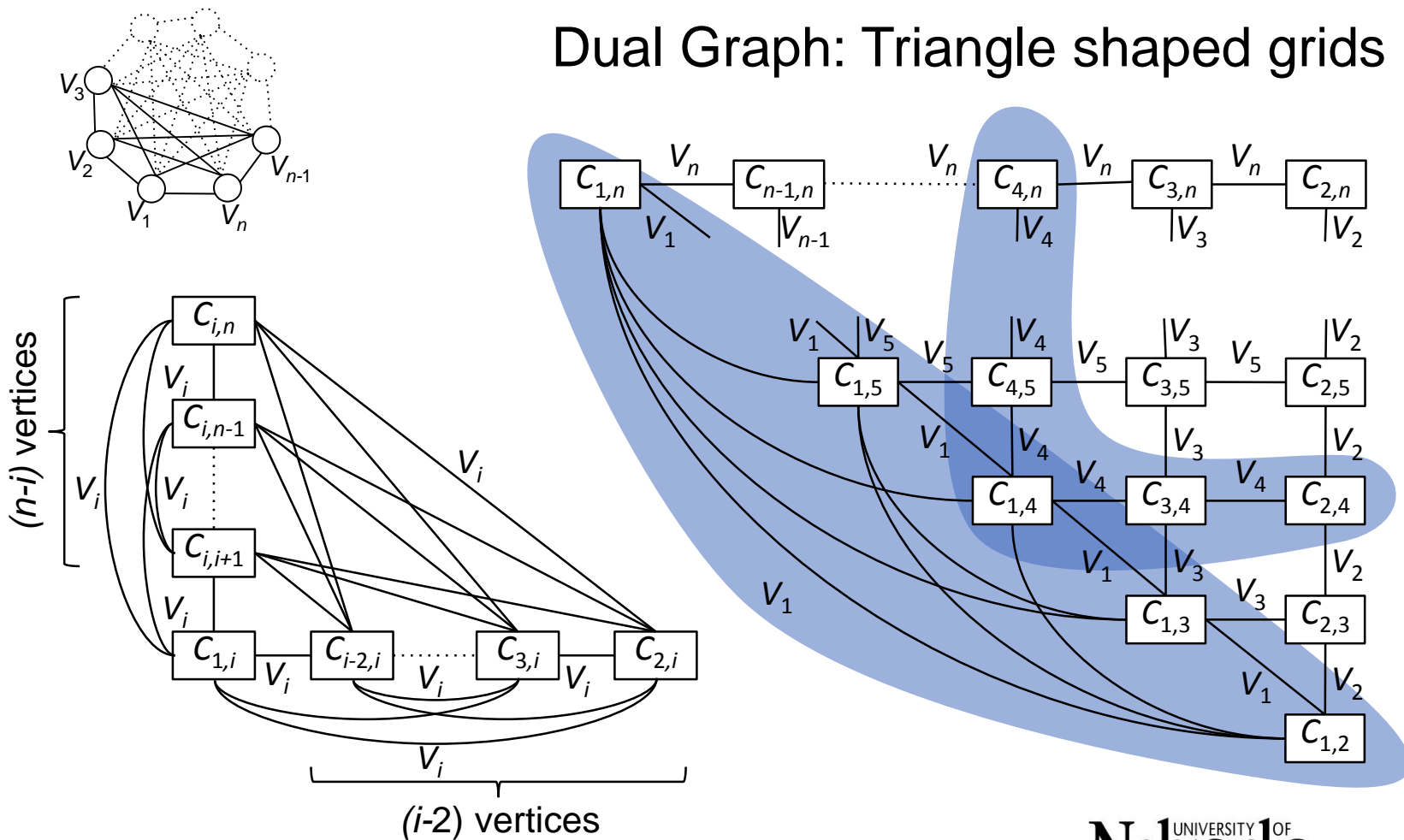
[Janssen+,1989]

Neighborhood Inverse Consistency

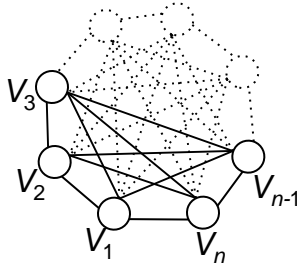
- Relational NIC [Woodward+ AAI 2011]
 - Reformulation of NIC [Freuder & Elfe, AAI 1996]
 - Defined for dual graph
 - Every tuple can be extended to a solution in its relation's neighborhood
 - Algorithm operates on dual graph & filter relations (not domains!)
 - Initially designed for non-binary CSPs
- How about RNIC on binary CSPs?
 - Impact of the structure of the dual graph
 - RNIC versus other consistency properties



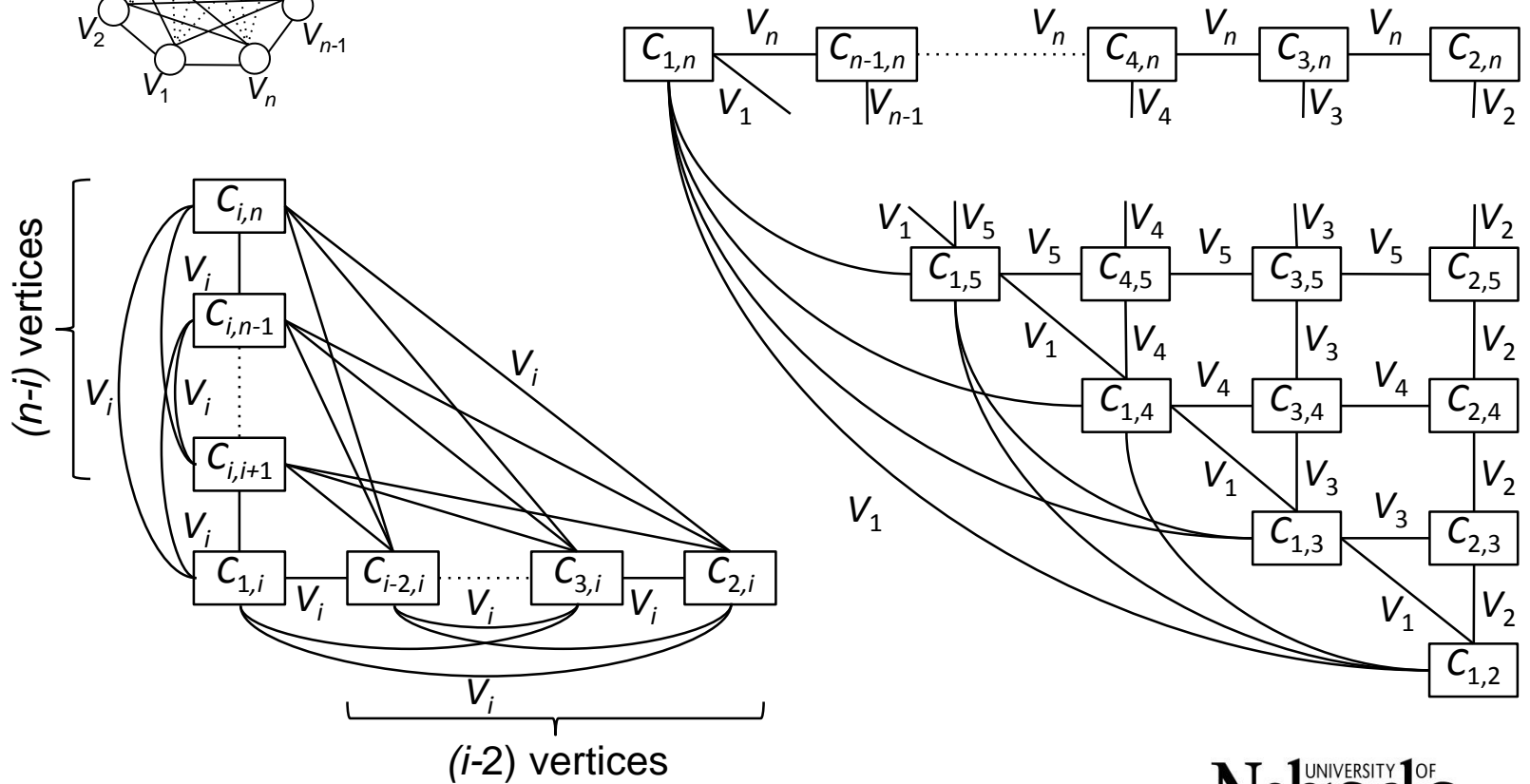
Complete Constraint Graph



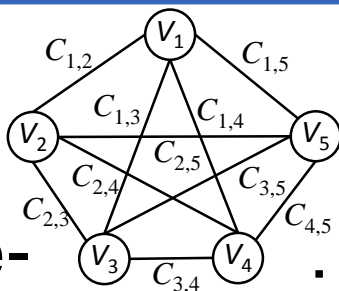
Minimal Dual Graph



Dual Graph: Triangle shaped grids



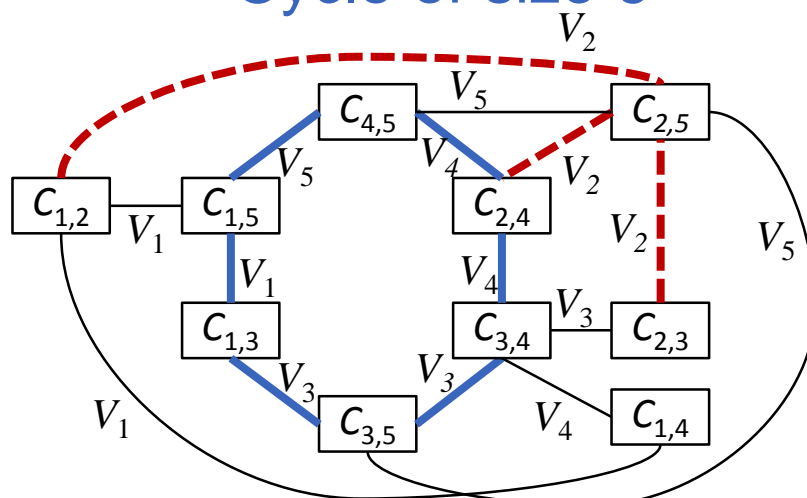
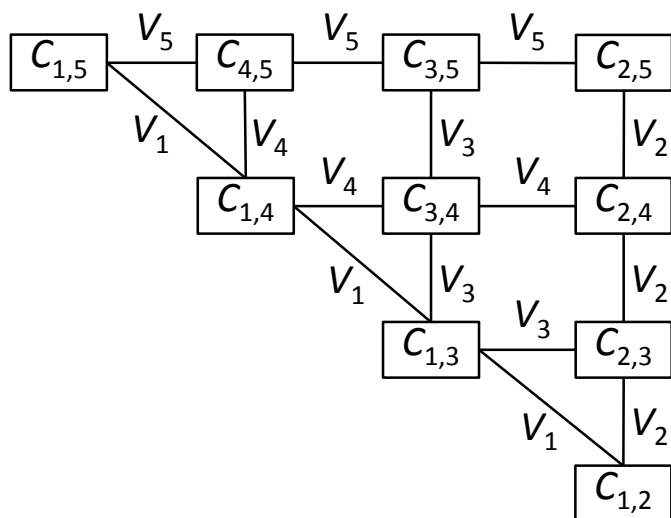
Minimal Dual Graph



... can be a triangle-shaped grid (planar)

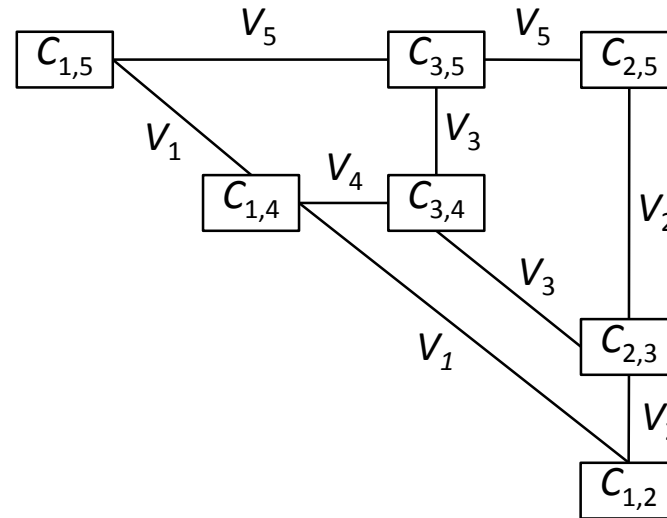
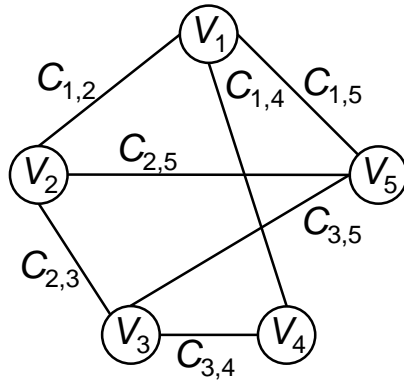
... but does not have to be

- Star on V_2
- Cycle of size 6



Non-Complete Constraint Graph

- Can still be a triangle-shaped grid
 - Have a chain of vertices
 - of length $\leq n-1$

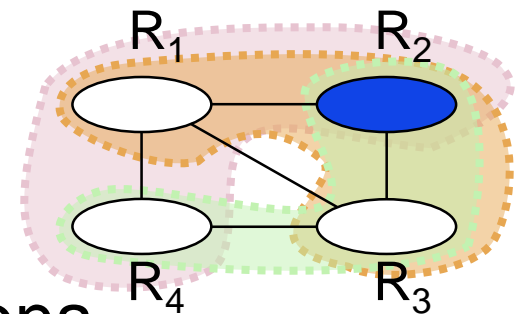


Impact on RNIC

On a binary CSP, RNIC enforced on the minimal dual graph (wRNIC) is never strictly stronger than $R(,3)C$.*

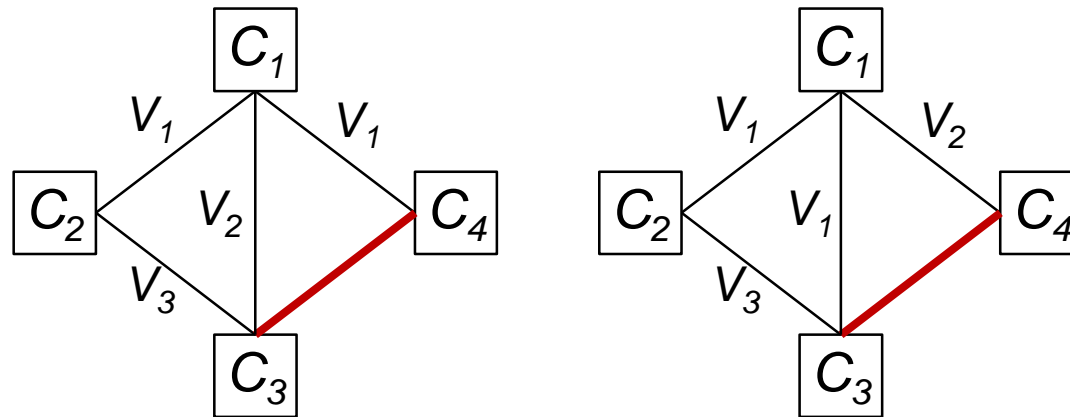
- $R(*,m)C$ ensures that subproblem induced on the dual CSP by every connected combination of m relations is minimal

[Karakashian+, AAI 2010]



wRNIC on Binary CSPs

- wRNIC can never consider more than 3 relations



- In either case, it is not possible to have an edge between C_3 & C_4 (a common variable to C_3 & C_4) while keeping C_3 as a binary constraint

NIC, sCDC, and RNIC not comparable

- NIC Property [Freuder & Elfe, AAAI 1996]

↳ Every **value** can be extended to a solution in its variable's neighborhood

- sCDC Property [Lecoutre+, JAIR 2011]

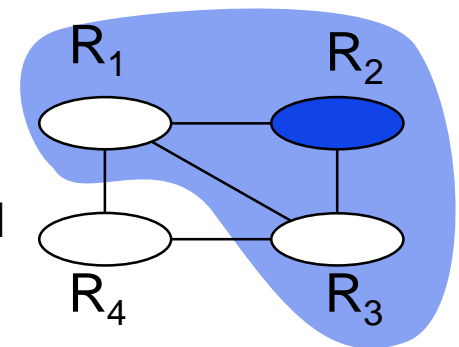
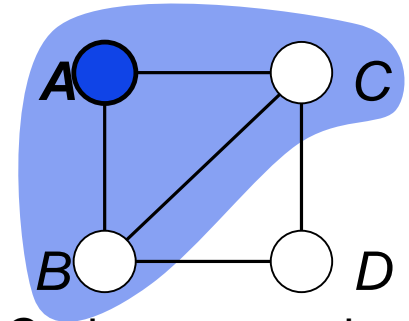
↳ An instantiation $\{(x,a),(y,b)\}$ is DC iff (y,b) holds in SAC when $x=a$ and (x,a) holds in SAC when $y=b$ and (x,y) in scope of some constraint. Further, the problem is also AC.

- RNIC Property [Woodward+, AAAI 2011]

↳ Every **tuple** can be extended to a solution in its relation's neighborhood

↳ wRNIC, triRNIC, wtriRNIC enforce RNIC on a minimal, triangulated, and minimal triangulated dual graph, respectively

↳ **seIRNIC** automatically selects the RNIC variant based on the density of the dual graph



Experimental Results (CPU Time)

Benchmark	# inst.	AC3.1	sCDC1	NIC	seIRNIC
CPU Time (msec)					
NIC Quickest					
bqwh-16-106	100/100	3,505	3,860	1,470	3,608
bqwh-18-141	100/100	68,629	82,772	38,877	77,981
coloring-sgb-queen	12/50	680,140	(+3) -	(+9) 57,545	634,029
coloring-sgb-games	3/4	41,317	33,307	(+1) 860	41,747
rand-2-23	10/10	1,467,246	1,460,089	987,312	1,171,444
rand-2-24	3/10	567,620	677,253	(+7) 3,456,437	677,883
sCDC Quickest					
driver	2/7	(+5) 70,990	(+5) 17,070	358,790	(+4) 185,220
ehi-85	87/100	(+13) 27,304	(+13) 573	513,459	(+13) 75,847
ehi-90	89/100	(+11) 34,687	(+11) 605	713,045	(+11) 90,891
frb35-17	10/10	41,249	38,927	179,763	73,119
RNIC Quickest					
composed-25-1-25	10/10	226	335	1,457	114
composed-25-1-2	10/10	223	283	1,450	88
composed-25-1-40	9/10	(+1) 288	(+1) 357	120,544	(+1) 137
composed-25-1-80	10/10	223	417	(+1) -	190
composed-75-1-25	10/10	2,701	1,444	363,785	305
composed-75-1-2	10/10	2,349	1,733	48,249	292
composed-75-1-40	7/10	(+1) 1,924	(+3) 1,647	631,040	(+3) 286
composed-75-1-80	10/10	1,484	1,473	(+1) -	397

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Experimental Results (BT-free, #NV)

Benchmark	# inst.	AC3.1	sCDC1	NIC	seIRNIC	AC3.1	sCDC1	NIC	seIRNIC
		BT-Free				#NV			
		NIC Quickest							
bqwh-16-106	100/100	0	3	8	5	1,807	1,881	739	1,310
bqwh-18-141	100/100	0	0	1	0	25,283	25,998	12,490	22,518
coloring-sgb-queen	12/50	1	-	16	1	91,853	-	15,798	91,853
coloring-sgb-games	3/4	1	1	4	1	14,368	14,368	40	14,368
rand-2-23	10/10	0	0	10	0	471,111	471,111	12	471,111
rand-2-24	3/10	0	0	10	0	222,085	222,085	24	222,085
		sCDC Quickest							
driver	2/7	1	2	1	1	3,893	409	3,763	3,763
ehi-85	87/100	0	100	87	100	1,425	0	0	0
ehi-90	89/100	0	100	89	100	1,298	0	0	0
frb35-17	10/10	0	0	0	0	24,491	24,491	24,491	24,346
		RNIC Quickest							
composed-25-1-25	10/10	0	10	10	10	153	0	0	0
composed-25-1-2	10/10	0	10	10	10	162	0	0	0
composed-25-1-40	9/10	0	10	9	10	172	0	0	0
composed-25-1-80	10/10	0	10	-	10	112	0	-	0
composed-75-1-25	10/10	0	10	10	10	345	0	0	0
composed-75-1-2	10/10	0	10	10	10	346	0	0	0
composed-75-1-40	7/10	0	10	7	10	335	0	0	0
composed-75-1-80	10/10	0	10	-	10	199	0	-	0

Conclusions

- Contributions
 - Apply RNIC to binary CSPs
 - Structure of dual graph & impact of RNIC
 - NIC, sCDC, and RNIC are incomparable
 - Empirically shown benefits of higher-level consistencies
- Future work
 - Study impact of the structure of the dual graph on (future) relational consistency properties
 - ‘Predict’ appropriate consistency property using information about the problem and its structure