Increasing Symmetry Breaking by Preserving Target Symmetries

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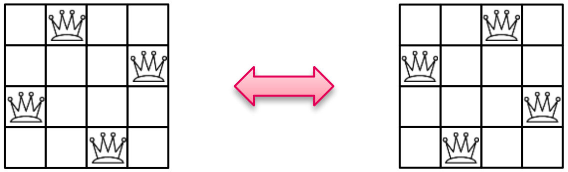
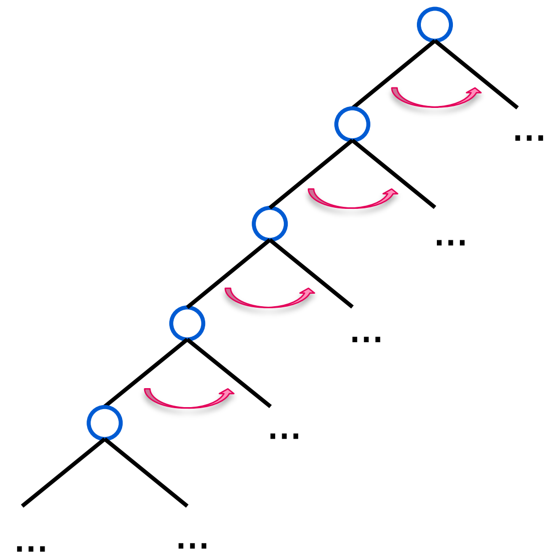
Overview

- A lot of CSPs are highly symmetric



- Breaking Symmetries in CSPs significantly reduce search tree size



- Two main methods to break symmetries
  - static symmetry breaking
  - dynamic symmetry breaking

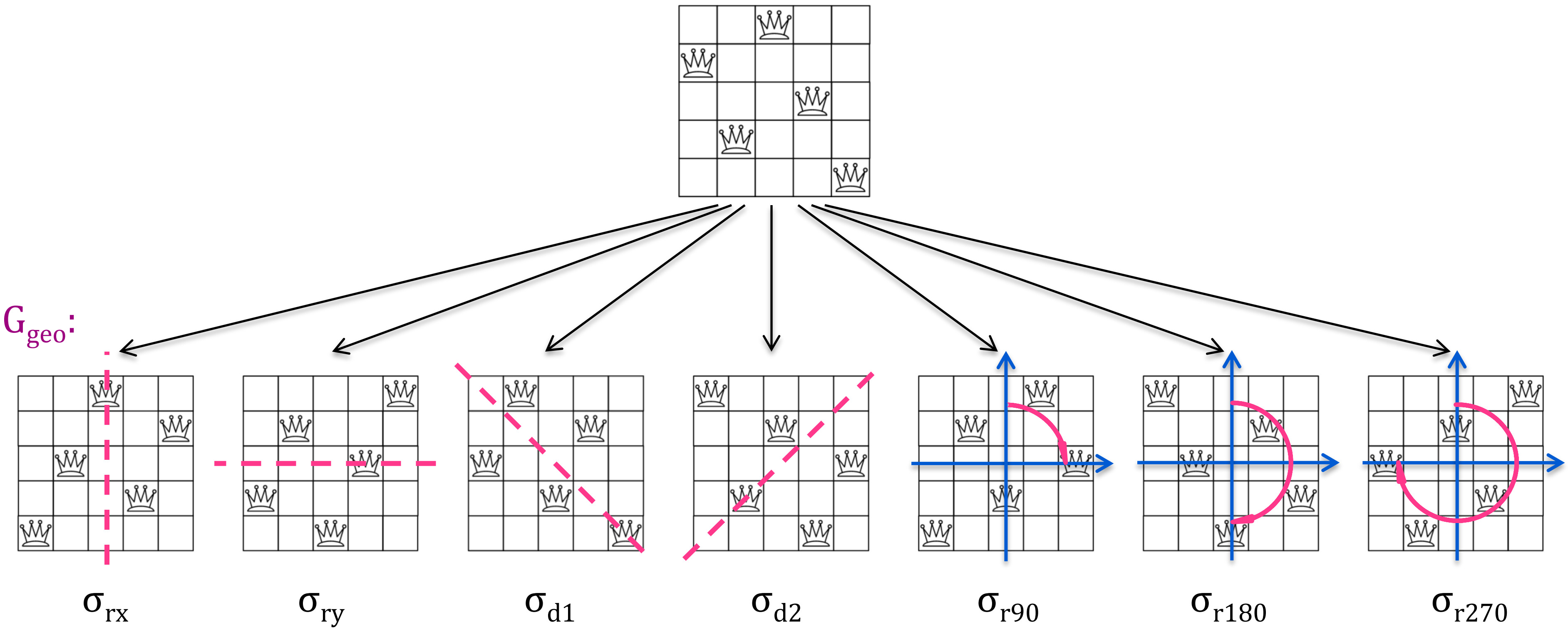
Overview

- Generally, exponential number of symmetries → exponential number of symmetry breaking constraints

$$\left\{ \begin{matrix} \sigma_1, \ \sigma_2, \ \sigma_3, \ \sigma_4, \ \sigma_5, \\ \dots \dots \\ \sigma_{n-4}, \ \sigma_{n-3}, \ \sigma_{n-2}, \ \sigma_{n-1}, \ \sigma_n \end{matrix} \right\} \longrightarrow \begin{matrix} C_{\sigma_1}, \ C_{\sigma_2}, \ C_{\sigma_3}, \ C_{\sigma_4}, \ C_{\sigma_5} \\ \dots \dots \\ C_{\sigma_{n-4}}, C_{\sigma_{n-3}}, C_{\sigma_{n-2}}, C_{\sigma_{n-1}}, C_{\sigma_n} \end{matrix}$$

- Partial Symmetry Breaking
  1. choose which symmetries to break [Jefferson and Petrie 2011]
  2. choose what constraints to use
     - preserving target symmetries

Symmetry



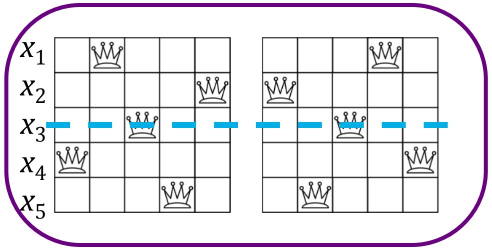
A symmetry on P (CSP Problem) for sol(P) is a bijective mapping σ on the v-vals(P) (variable value pairs) such that $sol(P)^{\sigma} = sol(P)$.

- <u>variable symmetry</u> σ — $(x_i = a_i)^{\sigma} \equiv x_{i\sigma} = a_i$
- <u>value symmetry</u> σ — $(x_i = a_i)^{\sigma} \equiv x_i = a_i^{\sigma}$
- <u>constraint symmetry</u> σ — preserve the set of constraints

Symmetry Breaking Constraints

vertical reflection σ_{ry}

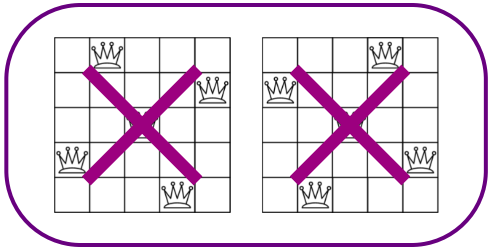
$$[x_1, x_2, x_3, x_4, x_5] \longrightarrow \sigma_{ry}([x_1, x_2, x_3, x_4, x_5])$$



Symmetry Breaking Constraints

vertical reflection σ_{ry}

$$[x_1, x_2, x_3, x_4, x_5] \rightarrow [x_5, x_4, x_3, x_2, x_1]$$



LEXLEADER [Crawford *et al.* 1996] constraint enforces lexicographical ordering on variable sequence:

$$X \leq_{\mathtt{lex}} \sigma(X)$$

$C_{\sigma_{ry}}$

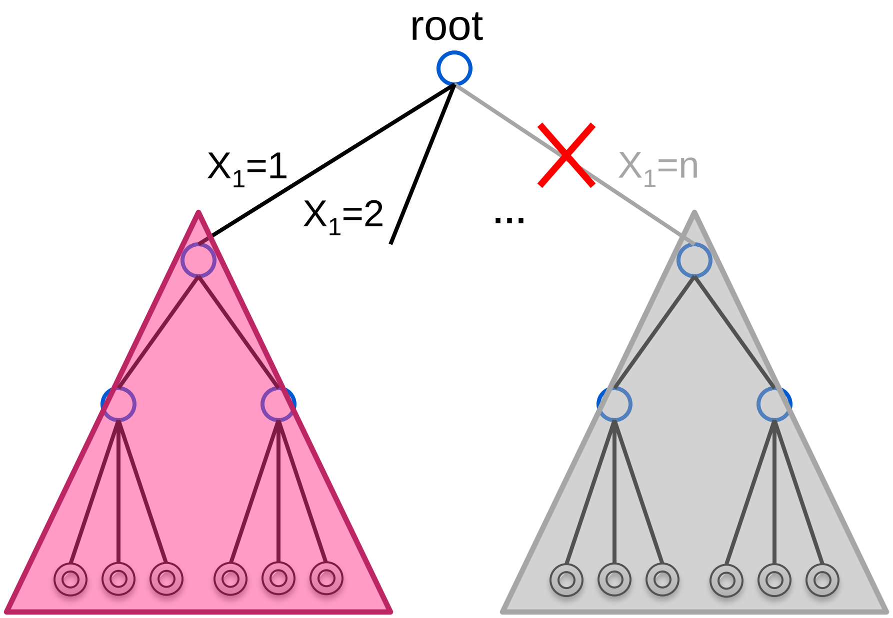
$$[x_1, x_2, x_3, x_4, x_5] \leq_{\mathtt{lex}} [x_5, x_4, x_3, x_2, x_1] \rightarrow x_1 < x_5$$

$$[x_5, x_4, x_3, x_2, x_1] \leq_{\mathtt{lex}} [x_1, x_2, x_3, x_4, x_5] \rightarrow x_5 < x_1$$

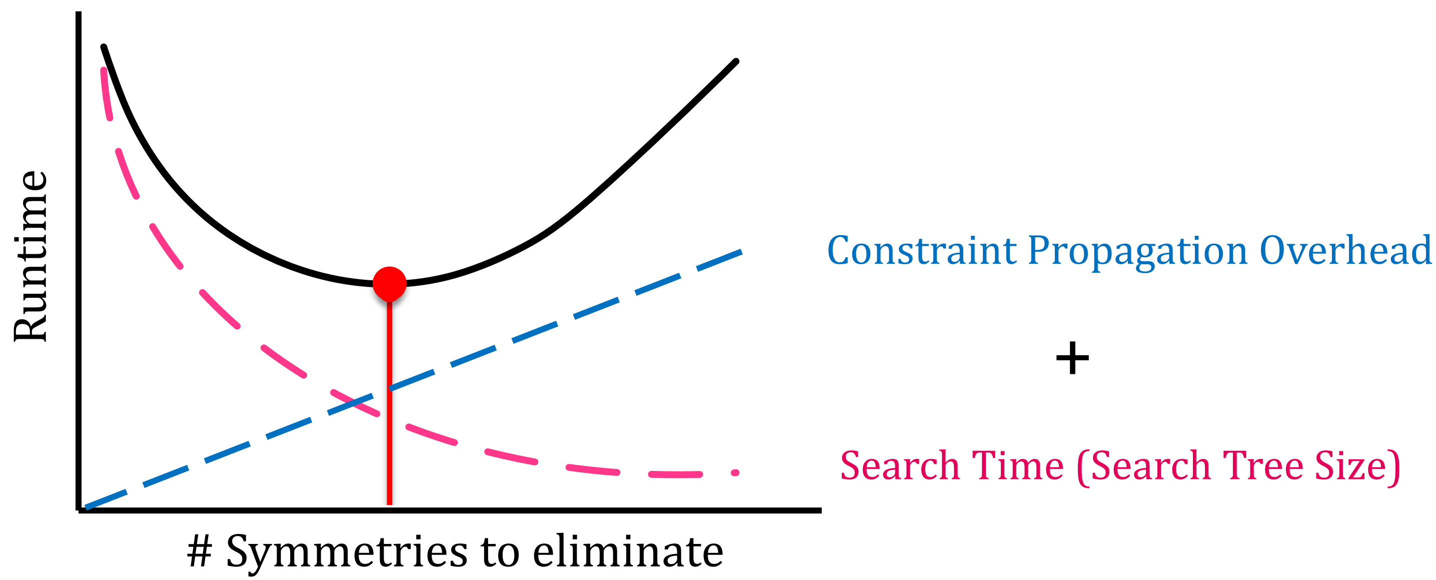
⋮

Symmetry Breaking in Search Tree

- The **goal** of breaking symmetries is to avoid the exploration of a search space with assignments that can be mapped by a representative in symmetry class via symmetry function.



Partial Symmetry Breaking
[McDonald and Smith 2002]



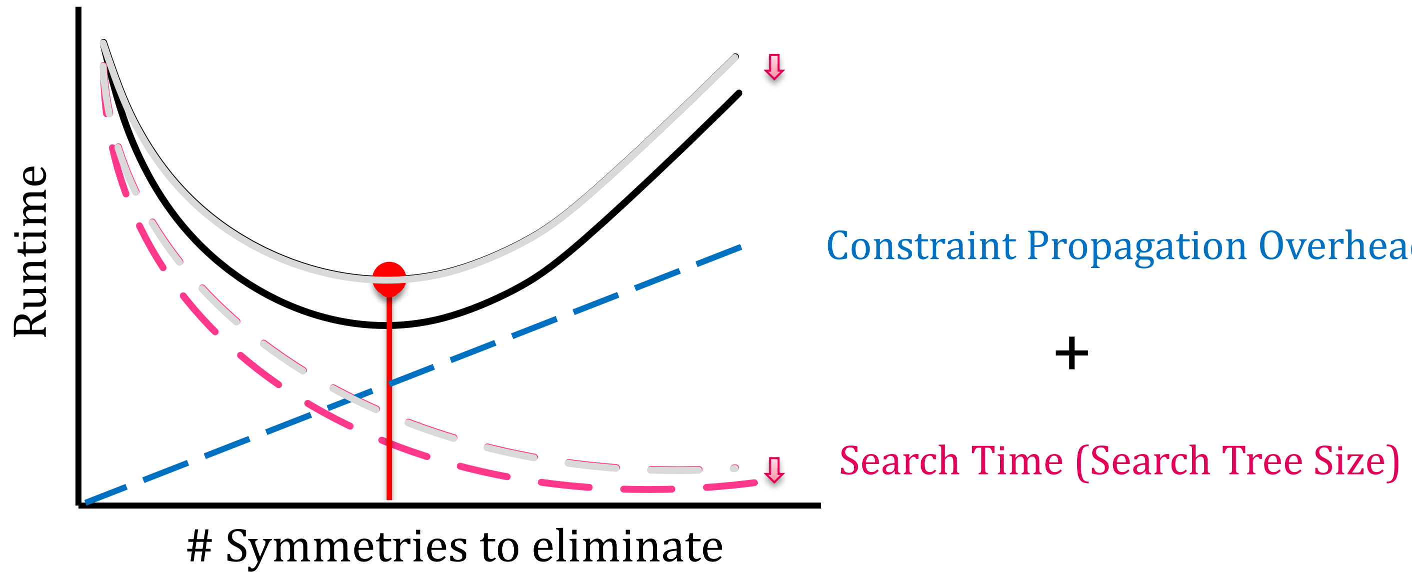
Target Symmetries!

$\{\sigma_1, \sigma_2, \sigma_3, \dots, \dots, \dots, \sigma_{n-1}, \sigma_n\} \supseteq \{\sigma_{i_1}, \sigma_{i_2}, \dots, \sigma_{i_m}\}$

$\{C_{\sigma_1}, C_{\sigma_2}, C_{\sigma_3}, \dots, \dots, \dots, C_{\sigma_{n-1}}, C_{\sigma_n}\} \supseteq \{C_{\sigma_{i_1}}, C_{\sigma_{i_2}}, \dots, C_{\sigma_{i_m}}\}$

Motivation and Goal

- Select a set of symmetry breaking constraints that aims to eliminate only target symmetries but is able to eliminate as much symmetries as possible.
  - similar overhead but turn out to prune more space finally



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$\{C_{\sigma_1}, C_{\sigma_2}, C_{\sigma_3}, \ldots, \ldots, \ldots, C_{\sigma_{n-1}}, C_{\sigma_n}\} \supseteq$ ~~$\{C_{\sigma_{i_1}}, C_{\sigma_{i_2}}, \ldots, C_{\sigma_{i_m}}\}$~~ $\{C'_{\sigma_{i_1}}, C'_{\sigma_{i_2}}, \ldots, C'_{\sigma_{i_m}}\}$

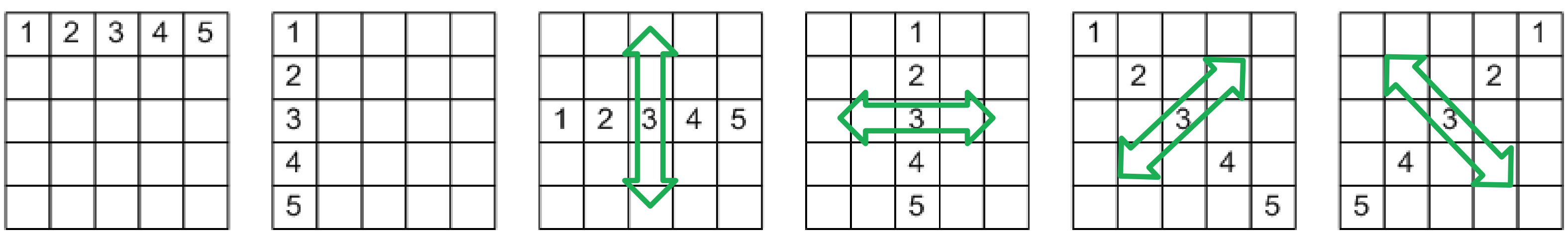
Symmetry Preservation - Definition

Definition 4.2: *Given a CSP $P = (V, D, C)$ with a symmetry σ. The symmetry* ***σ*** *is* ***preserved*** *by a set of symmetry breaking constraints* C^{sb} *iff σ is a symmetry of* $P' = (V, D, C \cup C^{sb})$.

Diagonal Latin Square

1	3	5	2	4
3	2	4	5	1
4	5	3	1	2
5	1	2	4	3
2	4	1	3	5

6 choices of value symmetry breaking constraints



none | none | vertical reflection | horizontal reflection | main diagonal reflection | minor diagonal reflection

Symmetry Preservation - Properties

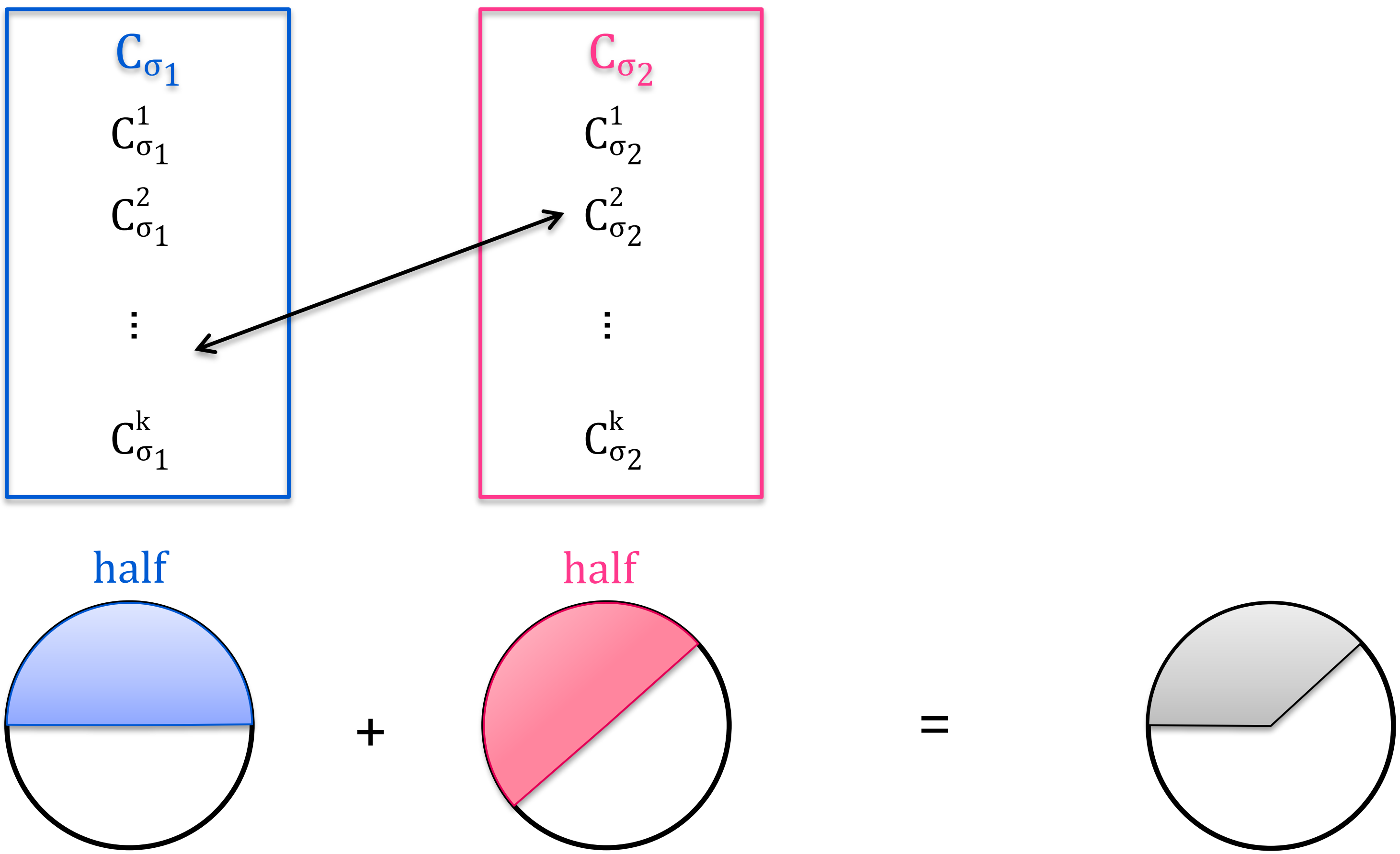
If C_{σ_1} preserve σ_2,

- **Theorem 4.3:** *Eliminating the target symmetries already eliminates their compositions*.
  - eliminate $\sigma_1, \sigma_2 \rightarrow$ eliminate $\sigma_1 \circ \sigma_2, \sigma_2 \circ \sigma_1$

- **Theorem 4.4:** *The combination of two sound symmetry breaking constraints is still sound*.
  - sound $C_{\sigma_1}, C_{\sigma_2} \rightarrow$ sound $C_{\sigma_1} \cup C_{\sigma_2}$

Why is Preservation good?

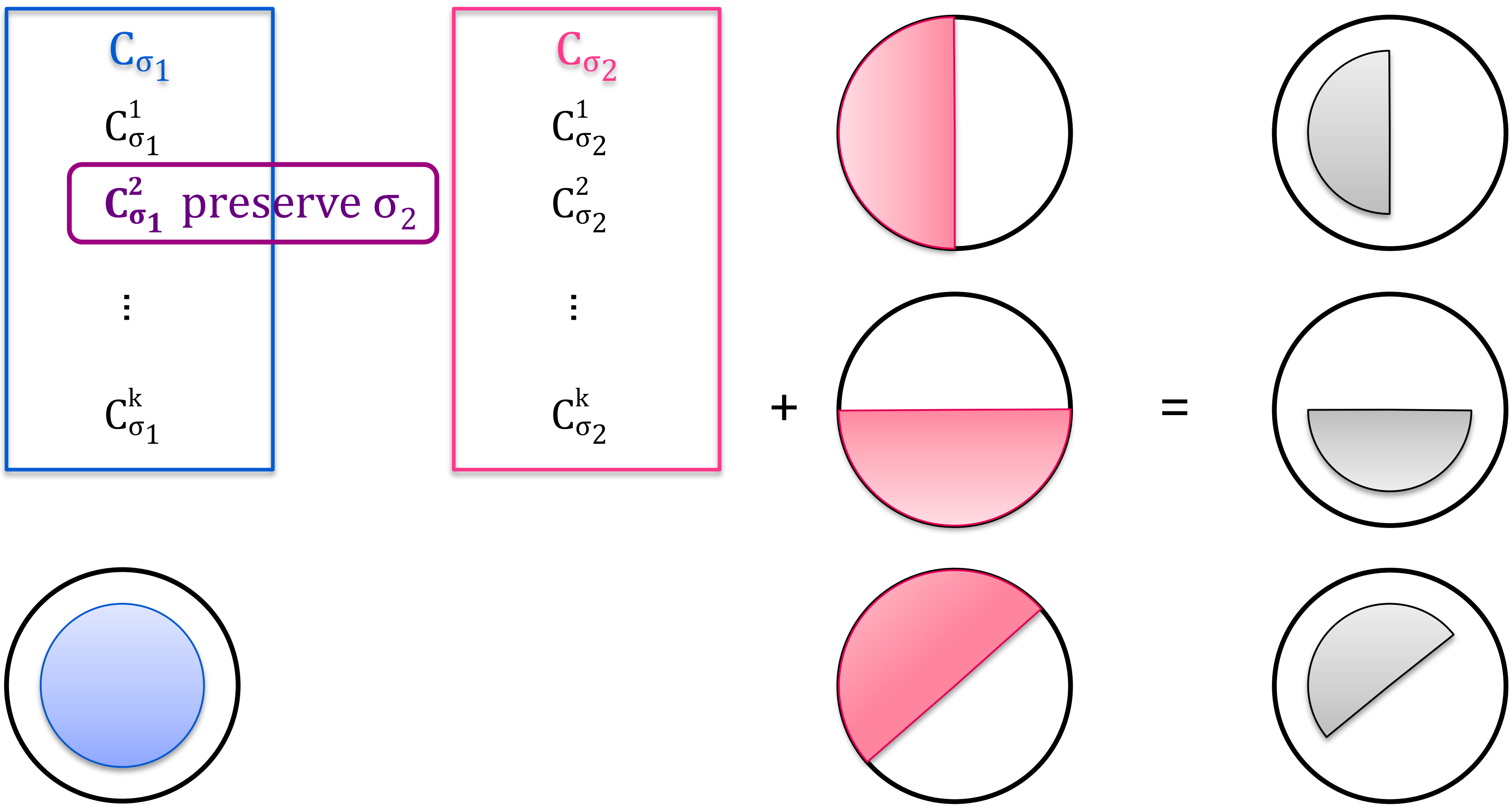
- Solution reduction of combining constraints



Why is Preservation good?

- guarantee good solution reduction

always ¼



Experimental Results

- 3GHz Intel Core2 Duo PC running Gecode-3.7.0
- # solutions, runtime (second), # fails

- Preserving Target Symmetries
  - Diagonal Latin Square Problem
  - NN-Queen Problem
  - Cover Array Problem (ILOG6.0)
  - Error Correcting Code - Lee Distance

Diagonal Latin Square (n)

- ROWWISE : $[X_{11}, X_{12}, \ldots, X_{1n}] = [1,2,\ldots,n]$
- Our Method : $[X_{11}, X_{22}, \ldots, X_{nn}] = [1,2,\ldots,n]$, $X_{1n} < X_{n1}$

n	ROWWISE			Our Method			
	#sol	time	#fails	#sol	time	#fails	
5	8	0.001	7	**4**	0.001	**1**	all solutions
6	128	0.029	3000	**64**	**0.004**	**652**	
7	171200	12.891	1413K	**85600**	**1.954**	**163K**	
8		0.002	140		**0.001**	**17**	one solution
9		40.04	4327K		**0.001**	**25**	
10		0.031	2025		**0.002**	**175**	
11		12052	1204124K		**0.005**	**339**	

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all solutions

half of the solution set size

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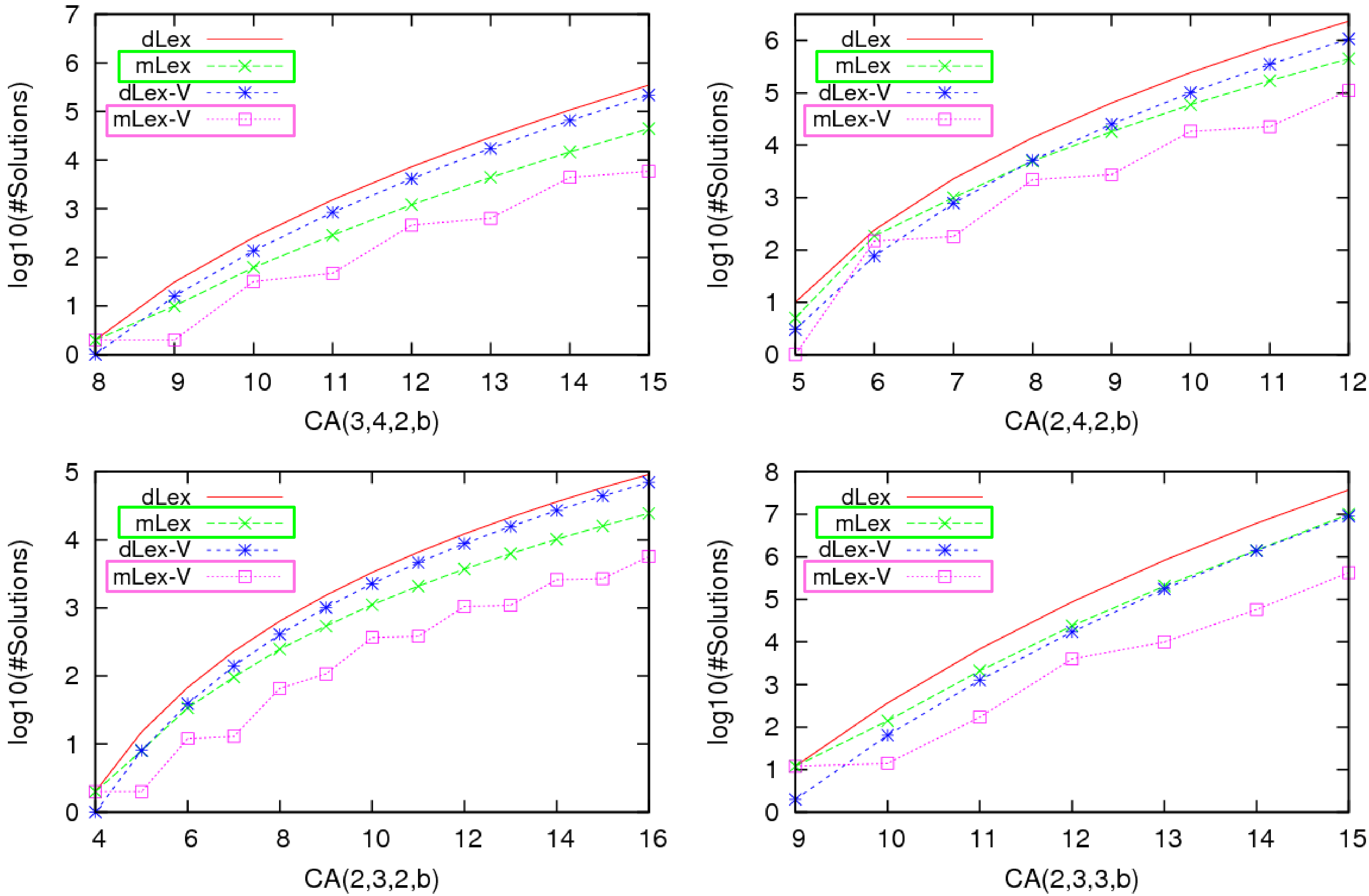
one solution

Cover Array Problem [Hnich *et al.* 2006]

- Target Symmetries:
  - row, column $\{G_{row}, G_{col}\}$
    - dLex [Flener *et al.* 2002]
    - **mLex** : $C_{G_{col}}$ [Frisch *et al.* 2003] preserves G_{row}
  - row, column, value $\{G_{row}, G_{col}, G_{val}\}$
    - dLex-V [Law and Lee 2004]
    - **mLex-V** : $C_{G_{val}}$ preserves $\{G_{row}, G_{col}\}$, $C_{G_{col}}$ preserves G_{row}

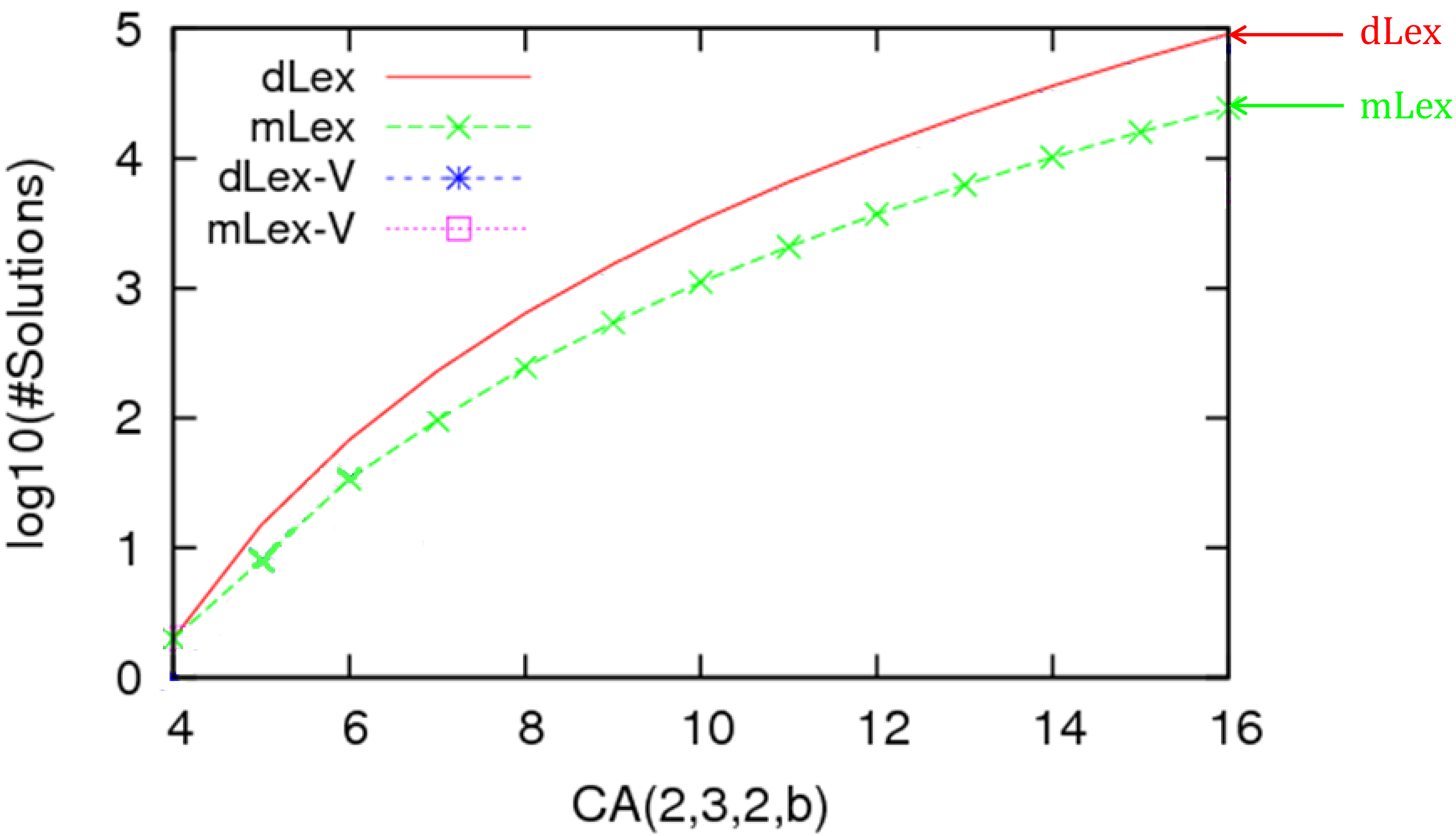
- Experiments are conducted with various problem sizes CA(t,k,g,b) .

Preserving Target Symmetries - CA ($\log_{10}$#solutions)



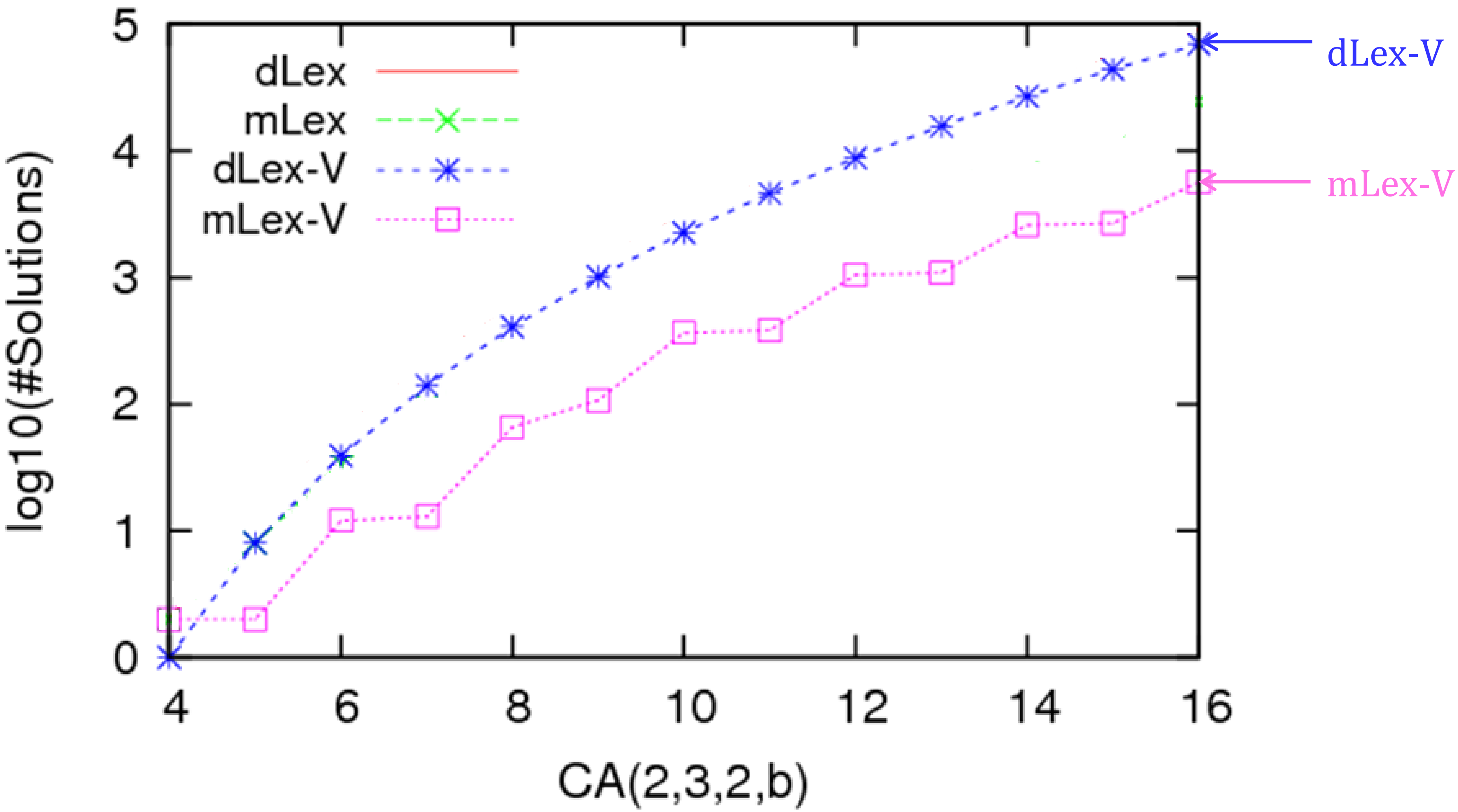
Cover Array Problem ($\log_{10}$#solutions)

Target Symmetries	Tradition	Our Method
row, column	dLex	mLex
row, column, value	dLex-V	mLex-V



Cover Array Problem ($\log_{10}$#solutions)

Target Symmetries	Tradition	**Our Method**
row, column	dLex	mLex
row, column, value	dLex-V	mLex-V



Cover Array Problem ($\log_{10}$#solutions)

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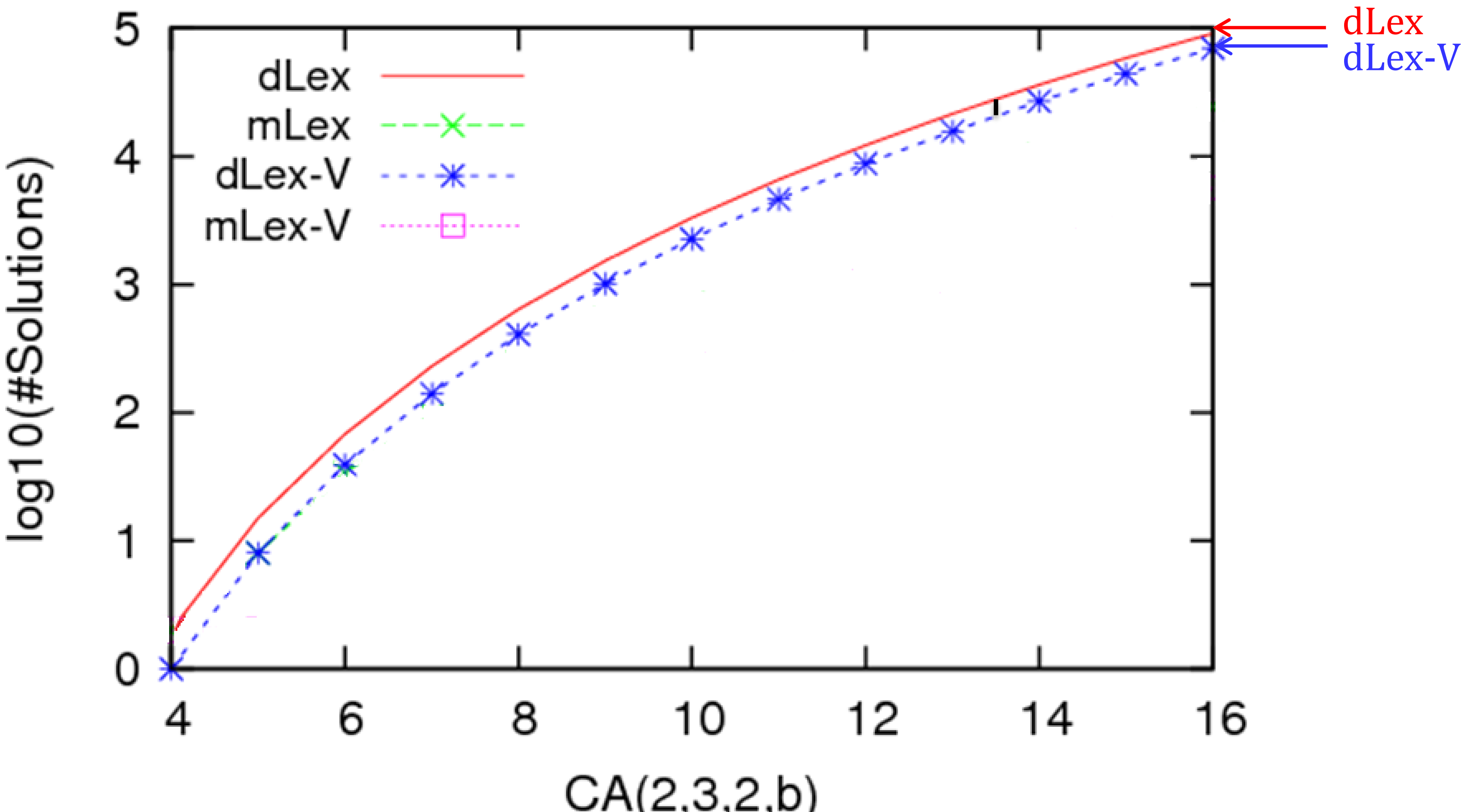
row, column	dLex	mLex
row, column, value	dLex-V	mLex-V



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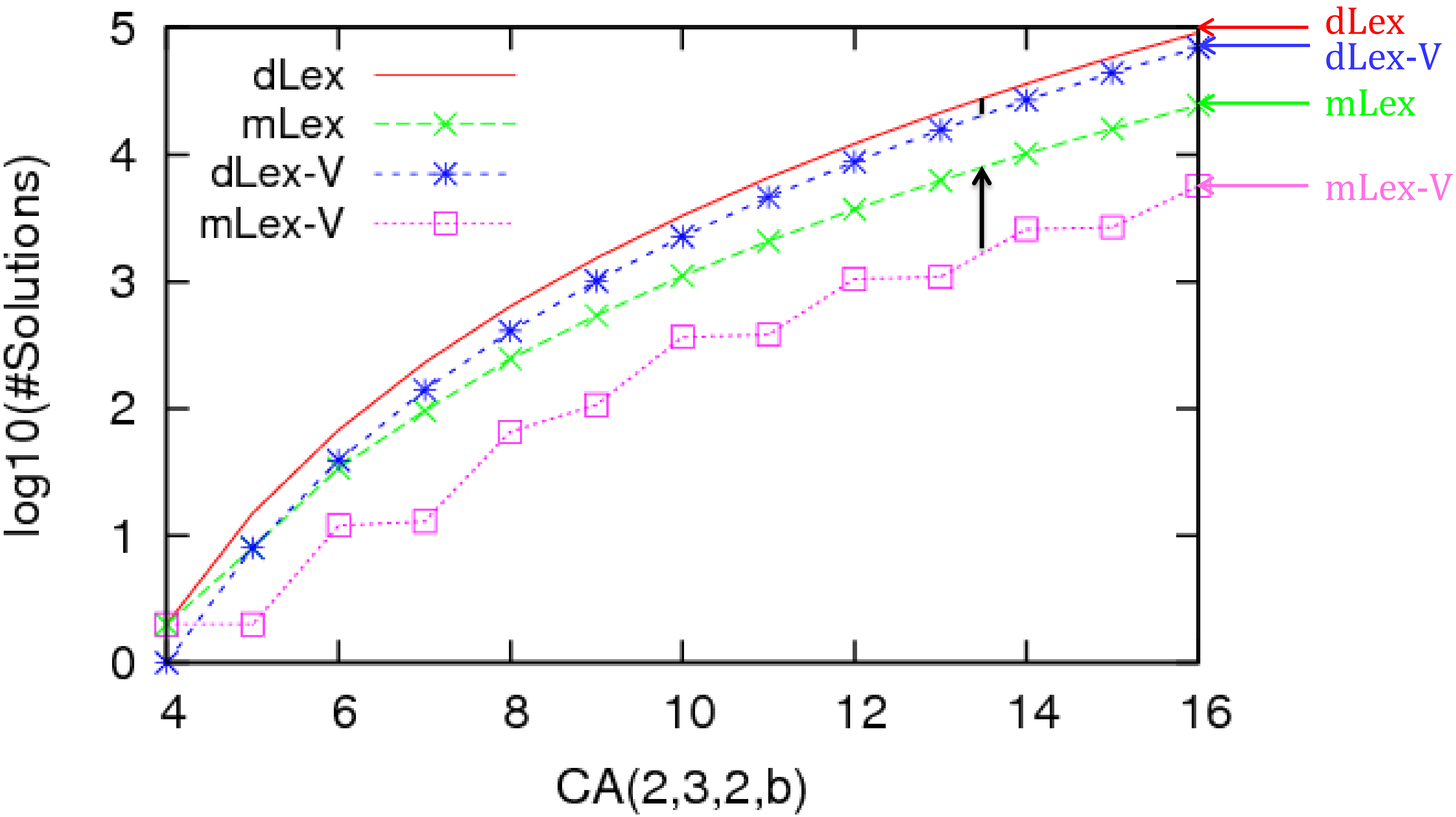

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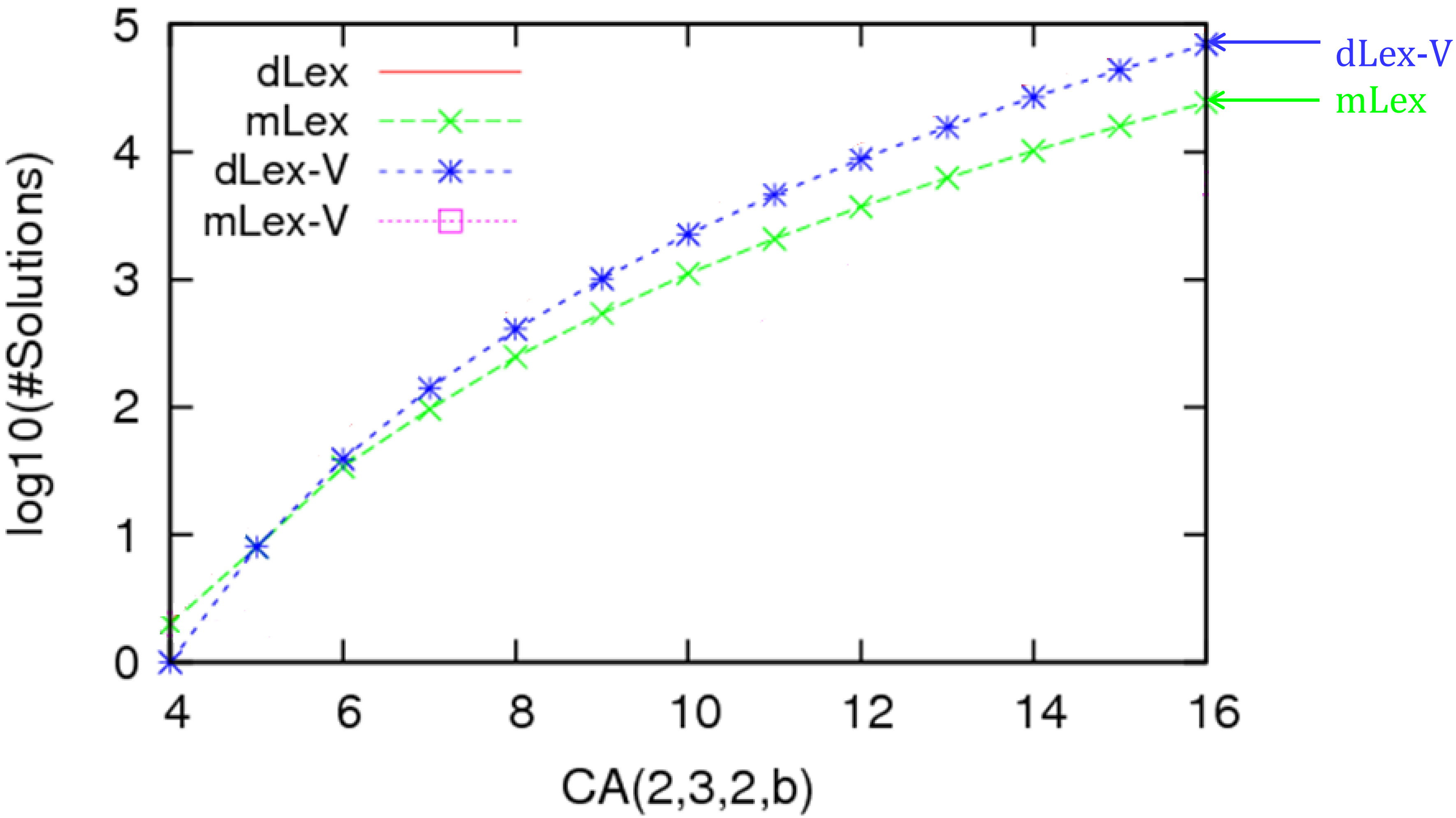
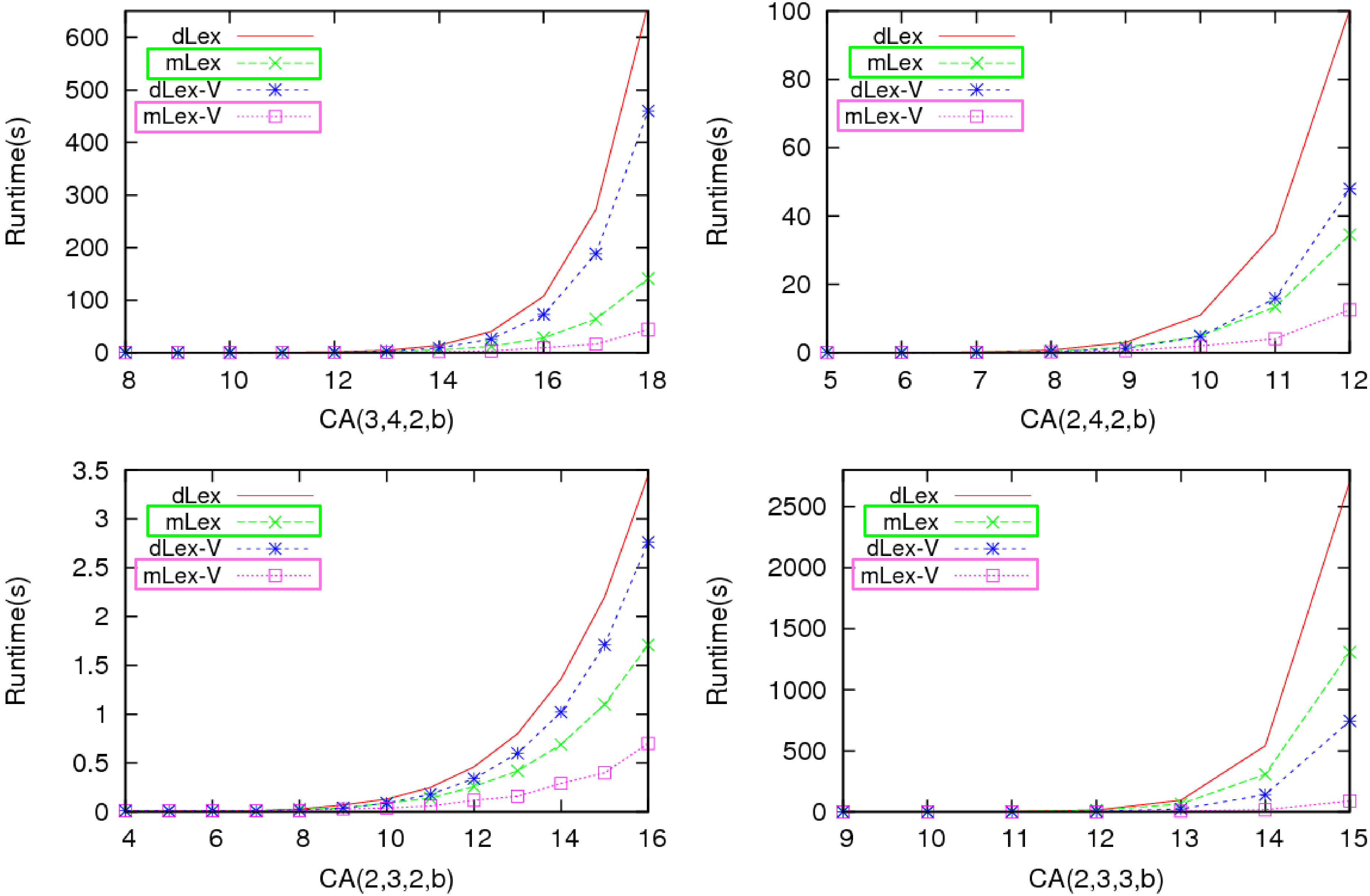
row, column	dLex	mLex
row, column, value	dLex-V	mLex-V



Preserving Target Symmetries - CA (runtime)



Concluding Remarks

1. formally define symmetry preservation

2. propose method to post symmetry breaking constraints for only target symmetries but they turn out to eliminate more symmetries

3. demonstrate the feasibility and efficiency of our proposals by experimental results

Future Work

- Combination with other approaches
  - choose better target symmetries [Jefferson and Petrie 2011]
  - model restart [Heller *et al.* 2008]
  - symmetries of sb constraints [Katsirelos and Walsh 2010]

- Automating the procedure of selecting constraints

THANK YOU!